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Mathematical Colloquia

Monday, May 20, 2019

17:15 h, Lecture Room 119

Dr. Tomasz Pelka, Polish Academy of Science

Planar rational cuspidal curves

Abstract:

Let E be a closed algebraic curve on a complex projective plane; and assume that E is homeomorphic to a projective line. The classification of such curves, up to a projective equivalence, is a classical open problem with interesting conuterparts in topology and symplectic geometry. The Coolidge-Nagata conjecture ('59), proved recently by Koras and Palka, asserts that every such curve is obtained from a line by some Cremona transformation. The known curves can in fact be constructed inductively. I will sketch this nice picture during my talk, including, as an example, two newly discovered families of curves. I will also use this construction to show that in the (most difficult) case of complement of log general type, a suprising rigidity holds: any such curve is uniquely determined, up to a projective equivalence, by the topology of its singular points.

This construction is posible due to existence of some specific lines, meeting E in at most two points. In the second part of my talk, I will explain the reason why such lines should exist in general: they come from extremal rays contracted by log MMP with half-integral coefficients.
The latter approach leads to the Negativity Conjecture, which asserts that the log Kodaira dimension of K+(1/2)D is negative. I will explain why this is the natural extension both of Coolidge-Nagata and some other, open problems concerning rigidity of (X,D). I will also report on our recent classification, up to a projective equivalence, of rational cuspidal curves satisfying this conjecture.

The aim of this talk is, on one hand, to advertise the log MMP modifications as a modern tool applicable in a much broader context, and on the other hand, to give new, elementary, but interesting examples of the rich geometry of planar curves and Cremona maps.