Mathematical Colloquia

Monday, 02 October 2023

17:15 h, lecture room B6 (ExWi)

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Polynomial optimization and sums of squares

Abstract: Polynomial optimization is a recent field, which emerged in the last two decades. It deals with optimization problems whose objective and constraints are polynomials (POPs). These problems have diverse applications, e.g., in operations research, discrete geometry, optimal control and theoretical computer science. However, they are computationally hard to solve exactly. Therefore, much attention has gone to finding tractable approximations for POPs.

The most successful of these rely on results from real algebraic geometry dating back to the work of Hilbert. Namely, they use sums of squares to verify nonnegativity of polynomials. By increasing the maximum degree of the sums of squares involved, this yields a hierarchy of bounds on the global optimum of the original POP, called the SOS-hierarchy. At a fixed level, these bounds may be computed by solving a semidefinite program, making them tractable.

Under mild conditions, the SOS-hierarchy converges asymptotically to the global optimum. A natural question is to understand the rate of this convergence, or equivalently, the quality of the approximations. In this talk, we discuss some recent advancements on this question in a variety of settings. We will use tools from real algebra, Fourier analysis and reproducing kernels.