Kähler and locally conformally Kähler metrics

Abstract: In many problems arising in differential geometry, an essential step is choosing a “good” metric. In complex geometry, we search for “good” metrics in the class of Hermitian metrics, which are Riemannian metrics compatible with the complex structure. In this class of Hermitian metrics, probably those which have the best properties are Kähler metrics. However, the existence of Kähler metrics on compact complex manifolds has a number of topological obstructions. Therefore, in non-Kähler geometry, one seeks to replace them with a suitable subclass of Hermitian metrics. One such subclass is represented by locally conformally Kähler (lcK) metrics, which were introduced by Vaisman. We present some properties of Kähler and lcK metrics on compact complex manifolds and then show how we can extend all the definitions to complex analytic spaces, and which results remain true in this more general setting.