

Mathematical Colloquia

Monday, 25 March 2024

17:15 h, lecture room B6 (ExWi)

Prof. Dr. Melanie Rupflin, Oxford University

Quantitative estimates for geometric variational problems

Abstract : Many interesting geometric objects are characterised as minimisers or critical points of natural geometric quantities such as the length of a curve, the area of a surface or the energy of a map.

For the corresponding partial differential equations and minimisation problems it is important to not only analyse the properties of exact solutions, but also to understand whether object that “almost solve the problem” provide good approximations of exact solutions.

In particular, it is natural to ask whether a function whose energy is very close to the energy minimum must also essentially “look like” a minimiser, and if so whether this “stability of minimisers” holds in a quantitative sense, i.e. whether one can bound the distance of an almost minimiser to the closest minimiser in terms of the energy defect.

In this talk we will discuss some of the ideas and challenges that appear in the study of this and related questions about the behaviour of almost solutions of PDE and variational problems. We will in particular consider the model problem of the classical Dirichlet energy of maps between spheres, where it has long been known that minimisers are given by meromorphic functions, but where the question of stability of minimisers has remained unclear until recently.