
Mathematical Colloquia

Monday, 15 April 2024

17:15 h, lecture room B6 (ExWi)

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On the values of the permanent function

Abstract : Two important functions in matrix theory, determinant and permanent, look very similar: $\det(A)$ is the sum over all permutations π of a $\text{sign}(\pi)$ times the product over all i of $a_{i,\pi(i)}$, and $\text{per}(A)$ is the same but without the factor $\text{sign}(\pi)$.

These functions are in the center of the investigations for several centuries because of their numerous applications and a number of open mathematical problems. For example, while the computation of the determinant can be done in a polynomial time, it is still an open question, if there are such algorithms to compute the permanent. Consequently, the question of the values that permanent can achieve, is actual. We will talk about recent results in this direction including the following two of our works.

In 1974 Wang [4, Problem 2] posed a question to find a decent upper bound for $|\text{per}(A)|$ if A is a square $(-1, 1)$ -matrix of rank k , see also Problem 6 in [3, Chapter 8.4]. In 1985 Kräuter [2] conjectured a certain upper bound. We prove the Kräuter's conjecture and thus obtain the complete answer to the Wang's question as well as characterized the matrices with the maximal possible permanent for each value of k .

We improve the Brualdi-Newman bound [1] for the consequent values of permanent of $(0, 1)$ matrices.

The talk is based on the joint works with M. Budrevich and K. Tarantin

[1] R.A. Brualdi, M. Newman, Some theorems on permanent, J. Res. Natl. Bur. Stand., Sect. B 69 (3) (1965) 159–163.

[2] A.R. Kräuter, Recent results on permanents of $(+1, -1)$ -matrices, Ber. No. 249, Berichte, 243-254, Forschungszentrum Graz, Graz, 1985.

[3] H. Minc, Permanents, Encyclopedia of Mathematics and its Applications, V. 6, 1978.

[4] E.T.H. Wang, On permanents of $(+1, -1)$ -matrices, Israel J. Math., 18 (1974) 353-361.