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Mathematical Colloquia

Monday, 19 March 2018

17:15 h, Lecture Room B 78

Prof. Dr. Franz Duzaar, University of Erlangen

A quantitative isoperimetric inequality in higher codimension

Abstract:

In this talk we explain a quantitative isoperimetric inequality in higher codimension. In a certain sense such an inequality can be viewed as a stability result (second order Taylor approximation) of Almgren's optimal isoperimetric inequality in higher codimension.

In particular, we prove that for any closed $(n-1)$-dimensional manifold Γ in $\mathbb{R}^{n+k}$ the following inequality

$$\mathbf{D}(\Gamma) \geq C\mathbf{d}^2(\Gamma)$$

holds true. Here, $\mathbf{D}(\Gamma)$ stands for the isoperimetric gap of Γ, i.e. the deviation in measure of Γ from being a round sphere. The quantity $\mathbf{d}(\Gamma)$ denotes a natural generalization of the classical Fraenkel asymmetry index of Γ to the higher codimension case. It measures – in a generalized sense – the distance of Γ from spheres of the same volume. As an immediate consequence one obtains an estimate of the isoperimetric quotient from below for sets which are close to the minimum (i.e. the spheres) of the form

$$\gamma(\Gamma) := \frac{\mathcal{H}^{n-1}(\Gamma)^{\frac{n}{n-1}}}{\mathcal{H}^n(Q(\Gamma))} \geq \gamma(S^{n-1}) + C\,\mathbf{d}^2(\Gamma).$$

Here, $Q(\Gamma)$ denotes an n-dimensional area minimizing surface with boundary Γ, and $\mathcal{H}^d$ the d-dimensional surface measure (Hausdorff measure).