Abstract:

A metric measure space is a complete, separable metric space equipped with a probability measure that has full support. Two such spaces are equivalent if they are isometric as metric spaces via an isometry that maps the probability measure on the first space to the probability measure on the second. The resulting set of equivalence classes can be metrized with the Gromov-Prohorov metric of Greven, Pfaffelhuber and Winter. We consider the natural binary operation that takes two metric measure spaces and forms their Cartesian product equipped with the sum of the two metrics and the product of the two probability measures. We show that the metric measure spaces equipped with this operation form a cancellative, commutative, Polish semigroup with a translation invariant metric. There is an explicit family of continuous semicharacters that is extremely useful for establishing that there are no infinitely divisible elements and that each element has a unique factorization into prime elements. We investigate the interaction between the semigroup structure and the natural action of the positive real numbers on this space that arises from scaling the metric.